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Bremsstrahlung (Cont'd):

To find the total energy loss rate of a high-energy particle due to bremsstrahlung, we should integrate $\frac{d^2W}{d\omega\, dt}$ over all frequencies. In practice, this means integrating to a cut-off frequency $\hbar\omega_{max} \simeq E$. We then find:

$$\frac{dE}{dt} \simeq -\frac{16 Z^2 e^6 \gamma n}{3 c^3 m_e^2 v} \int_0^{\omega_{max}} \ln\Lambda \, d\omega$$

Here $\ln\Lambda = \ln\left(\frac{b_{max}}{b_{min}}\right)$, where appropriate values of b_{max}, b_{min} depend on the frequency. It is convenient to write:

$$g(v,\omega) = \frac{\sqrt{3}}{\pi} \ln\Lambda$$

$g(v,\omega)$ is a correction factor, which is called the Gaunt factor.

In the non-relativistic limit $\gamma \simeq 1$ and $\omega_{max} \simeq \frac{1}{2}\frac{m_e v^2}{\hbar}$.

This results in:

$$\frac{dE}{dt} = -(\text{Const.})\, Z^2 n v \Rightarrow \frac{dE}{dt} \propto -E^{\frac{1}{2}}$$

Therefore, bremsstrahlung loss is more efficient at higher energies. It can also be seen that a factor of $(m_e)^{-3/2}$ appears in $\frac{dE}{dt}$ in the non-relativistic limit. As expected, it implies that the lighter the charged particle, the more efficient its energy loss via bremsstrahlung will be.

An important point to note is that $\frac{dE}{dt}$ is Lorentz invariant. Hence, the result obtained from calculations in the frame of the moving electron is also valid in the lab frame.

In the relativistic limit, the energy loss rate is given by the Bethe-Heitler formula derived from the full relativistic quantum treatment. It takes various effects like the Fermi-Thomas shielding effect into account. We just give the expression in the relativistic limit:

$$\frac{dE}{dt} = -\frac{8Z(Z+1.3)e^6 n}{3 m_e^2 c \hbar} E\left[\ln\left(\frac{183}{Z^{1/3}}\right) + \frac{1}{8}\right]$$

Note that $\frac{dE}{dt} \propto -E$ in this case.

Thermal Bremsstrahlung:

Consider a flux of electrons passing by ions, where n_e and n are the respective number densities. The frequency-dependent emissivity of the medium is given by:

$$\frac{d^3W}{d\omega\, dt\, dV} = \frac{16\pi e^6}{3\sqrt{3} c^3 m_e^2 v} n_e n Z^2 g(v,\omega)$$

Here we have assumed the non-relativistic regime. If electrons have a thermal distribution, then their number density follows the Maxwell-Boltzmann distribution:

$$n_e(v)\, dv = 4\pi n_e \left(\frac{m_e}{2\pi kT}\right)^{3/2} v^2 \exp\left(\frac{-m_e v^2}{2kT}\right) dv$$

Thermally-averaged emissivity will then be:

$$\varepsilon_{(\nu)} \equiv \frac{d^3W}{d\nu\, dt\, dV} = \frac{32\pi^2 Z^2 e^6 n_e n}{3\sqrt{3} c^3 m_e^2} \mathrm{Int}(\nu, T)$$

Here:

$$Int(\nu, T) \equiv \frac{\int_{v_{min}}^{\infty} g(\nu, v)\, v^2 \exp\left(-\frac{m_e v^2}{2kT}\right) dv}{\int_0^{\infty} v^2 \exp\left(\frac{-m_e v^2}{2kT}\right) dv}$$

Note the lower limit in the integral in the numerator. It is due to the fact that a minimum speed $v_{min} = \left(\frac{2h\nu}{m_e}\right)^{\frac{1}{2}}$ is required to emit a photon with frequency ν.

These integrals can be evaluated, which gives rise to the following result written in terms of a thermally averaged Gaunt factor:

$$\epsilon_{(\nu)} = 6.8 \times 10^{-38}\, n_e n\, Z^2\, T^{-\frac{1}{2}} \exp\left(-\frac{h\nu}{kT}\right) g(\nu, T) \text{ erg cm}^{-3}\text{s}^{-1}\text{Hz}^{-1}$$

At frequencies $h\nu \ll kT$, $g(\nu, T)$ has only a logarithmic dependence on ν. Suitable forms of $g(\nu, T)$ at radio and X-ray frequencies are:

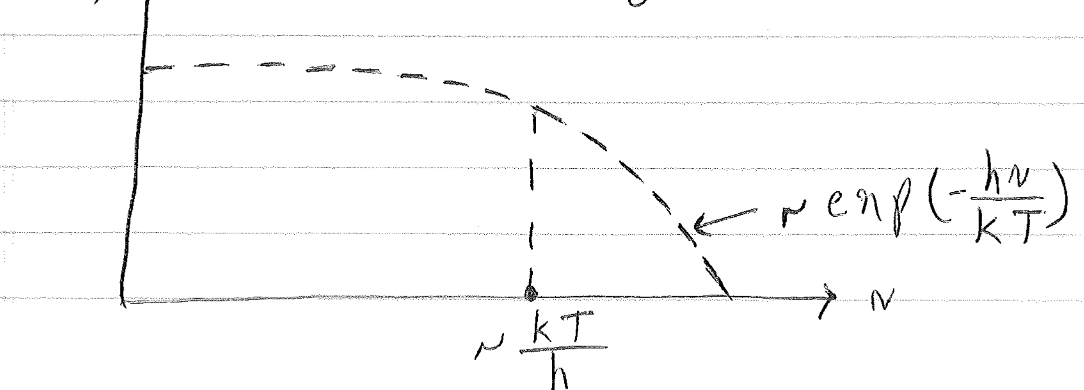
Radio: $$g(\nu, T) = \frac{\sqrt{3}}{2\pi}\left[\ln\left(\frac{2k^3T^3}{\pi^3 m_e e^4 \nu^2 Z^2}\right) - \gamma^{\frac{1}{2}}\right]$$

X-ray: $g_{(\nu,T)} = \frac{\sqrt{3}}{\pi} \ln\left(\frac{kT}{h\nu}\right)$

$\gamma = 0.577...$ is Euler's constant.

For the vast majority of astrophysical thermal plasmas, $1 \lesssim g_{(\nu,T)} \lesssim 5$.

Due to the mild frequency-dependence of $g_{(\nu,T)}$ at low frequencies, it is seen that the spectrum is flat for photon energies much smaller than the thermal energy kT. As we will see later, such a profile is unique among various emission mechanisms thereby providing a distinct observational signature. The spectrum turns over at $h\nu \sim kT$ dropping off exponentially at higher energies:



Note that the existence of a "Bremsstrahlung knee" provides the means of measuring the temperature of the emitting plasma. For example, the spectrum of the intergalactic gas in the Perseus cluster of galaxies as observed in the X-ray waveband by the HEAO-A2 experiment can be used to derive a temperature $T = 7.5 \times 10^7$ K of the emitting gas.

So far, we have assumed that the photons produced via bremsstrahlung leave the system unaffected. In other words, the spectrum is the optically thin bremsstrahlung spectrum. But this is not always the situation encountered in high-energy astrophysics. The medium may not be thin enough for radiation to scape without further interaction with the plasma. If the mean free path of photons l for scattering (or reabsorption) is large compared with the size R of the system, then we will have an optically thin emission. Since $l \simeq \frac{1}{n_e \sigma}$ (σ being the cross-section for scattering), this happens when $\tau \equiv n_e \sigma R \ll 1$, where τ is called the optical depth. In the other limit when $\tau \gg 1$, the photons undergo numerous scatterings before leaving the system. Since particles in the medium have thermal distribution, the radiation will emerge with a blackbody spectrum in this case, which has a prominent bump at $h\nu \sim kT$.